Mesoscopic Hall effect driven by chiral spin order

Jun-ichiro Ohe[1], Tomi Ohtsuki[2], and Bernhard Kramer[3]
[1]*I. Institut für Theoretische Physik, Universtität Hamburg, Jungiusstrasse 9, 20355 Hamburg, Germany*
[2]*Department of Physics, Sophia University, Kioi-cho 7-1, Chiyoda-ku, Tokyo 102-8554, Japan*
[3]*School of Engineering and Science, International University Bremen, Campus Ring 1, 28757 Bremen, Germany*
(Dated: February 3, 2008)

A Hall effect due to spin chirality in mesoscopic systems is predicted. We consider a 4-terminal Hall system including local spins with geometry of a vortex domain wall, where strong spin chirality appears near the center of vortex. The Fermi energy of the conduction electrons is assumed to be comparable to the exchange coupling energy where the adiabatic approximation ceases to be valid. Our results show a Hall effect where a voltage drop and a spin current arise in the transverse direction. The similarity between this Hall effect and the conventional spin Hall effect in systems with spin-orbit interaction is pointed out.

PACS numbers: 72.25.Dc, 72.10.Fk, 73.23.-b

Recent research on the anomalous Hall effect has shown that the spin chirality of a local spin system induces a Hall conductance via exchange coupling [1, 2, 3, 4]. The anomalous Hall effect can be seen in ferromagnetic metallic systems where the time reversal symmetry (TRS) is broken. When TRS is preserved, the spin current in a transverse direction is driven by a longitudinal voltage drop. Such a spin current, the so-called spin Hall current [5, 6, 7], can be seen in semiconductor systems with a spin-orbit interaction. Both anomalous and spin Hall effects were originally expected in bulk systems where the gauge field related to monopoles in momentum space plays a crucial role. The spin Hall effect can also be seen in mesoscopic 2-dimensional samples with Rashba spin-orbit interaction [6]. Numerical calculations, using both the Kubo and Landauer-Buttiker formulae predict the spin Hall effect. Note that the Landauer-Buttiker formula [8, 9] does not explicitly assume a local electric field inside the sample [10], while such a field seems to be essential for explaining the conventional spin Hall effect [5].

Recently, it has been shown that a Hall conductance is expected in mesoscopic systems such as dilute magnetic semiconductors with artificial magnetic structures [11, 12]. The spin of the conduction electron couples to the local magnetic moment via an exchange interaction. The Hall conductance is determined in such a well ordered magnetic system by using a local gauge transformation and the adiabatic approximation in which only a majority spin component is considered. This approximation changes the symmetry of the system from SU(2) to U(1). However, the minority spin of conduction electrons cannot be neglected when the exchange coupling energy is comparable to the Fermi energy as in magnetic semiconductors [13].

In this letter, we show that mesoscopic systems with internal chiral magnetic order exhibit Hall effects in such a way that both the charge and the spin Hall effects occur simultaneously. We consider a 2-dimensional electron system that interacts with local spins via exchange coupling. The local spins have a vortex structure with a finite out-of-plane component that determines the spin chirality. We assume that the Fermi energy of the conduction electron is comparable to the exchange coupling energy as in some magnetic semiconductors. In such energy region, the adiabatic approximation and the U(1) mean field theory [4] that explains the anomalous Hall effect cannot be applied. We calculate the spin-resolved Hall conductance numerically by using the recursive Green function method [14, 15]. Our numerical results show that a Hall voltage is induced when the system has spin chirality. Furthermore, a spin Hall current can be observed even if the system does not have a spin chirality. This spin Hall current does not require an electric field inside the system unlike conventional spin Hall effects in bulk systems with spin-orbit interaction [5]. We mention that the system considered here is related to a 2-dimensional spin-orbit system for which the spin Hall effect has been reported. We show that the previously reported spin Hall effects [16, 17] obtained from the

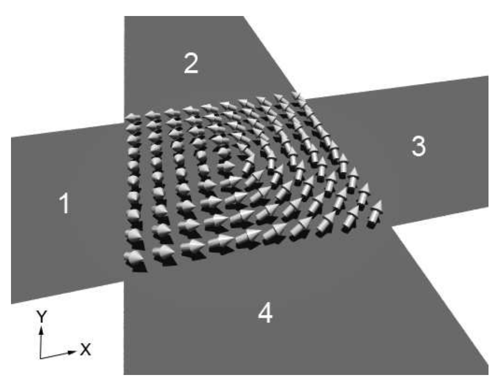

FIG. 1: Schematic of proposed 4-terminal system, including chiral magnetic structure. Labels 1-4 indicate leads that are free from randomness, local magnets, or spin-orbit interaction.

Landauer-Buttiker formula are similar to the Hall effect presented here. We investigate the coupling constant dependence of the spin Hall conductances for these systems. Both of the spin Hall conductances oscillate when increasing the exchange or spin–orbit coupling strength and show linear dependences in the weak coupling regime.

We consider a 2-dimensional electron system with exchange interaction [18],

$$H=-t\sum_{<i,j>,\sigma,\sigma'}c^{+}_{i\sigma}c_{j\sigma'}-J\sum_{i,\sigma,\sigma'}c^{+}_{i\sigma}\boldsymbol{\sigma}c_{i\sigma'}\cdot\mathbf{S}(x,y),\quad(1)$$

with the nearest-neighbor hopping parameter $t=\hbar^2/2m^*a^2$ (m^* effective mass, a lattice parameter). Operator $c^{+}_{i\sigma}(c_{i\sigma})$ creates (annihilates) an electron of spin σ at lattice site i, $\boldsymbol{\sigma}$'s are the Pauli matrices, and $J(>0)$ is the exchange coupling constant. The local spin $\mathbf{S}(x,y)$ has the geometry of a vortex in the x-y plane in addition to the uniform S_z-component

$$\mathbf{S}(x,y)=S(\cos\phi(x,y)\sin\theta,\sin\phi(x,y)\sin\theta,\cos\theta).\quad(2)$$

Here, S is the modulus of the local spin, $\phi(x,y)=-\tan^{-1}(y/x)$, such that the center of the vortex is located at the origin. We assume that the dynamics of the local spins is much slower than that of the conduction electrons, and treat the local spins as static. A schematic of the system is shown in Fig. 1 in which we assume 4-terminal geometry. The leads are labeled as 1-4 and the $+x(y)$-direction is set to the direction from the lead 1(4) to 3(2). We define the chirality of the local spin system,

$$\mathrm{Ch}_{ijkl}\equiv E_{ijk}+E_{ikl},\quad(3)$$

where $E_{ijk}=\mathbf{S}_i\cdot(\mathbf{S}_j\times\mathbf{S}_k)$ for a plaquette of a square lattice labeled as (i,j,k,l) counter–clockwise. Fig. 2 shows the distribution of the spin chirality for $\theta=0.3\pi$. The system shows large spin chirality near the center of the vortex. The value of the spin chirality changes the sign when the sign of the S_z-component changes. Obviously, the local spin system does not have a chirality, $\mathrm{Ch}_{ijkl}=0$, for $\theta=0,\pi/2,\pi$. As in the case of the bulk anomalous Hall effect due to spin chirality [4], we expect that the Hall effect can be obtained except for these values of θ.

We calculate the spin-resolved transmission amplitudes by using the recursive Green function method [14, 15]. By employing the Landauer-Buttiker formula, we assume that the net current of the leads 2 and 4 is zero. The current of lead I_l is $I_l=\sum_\sigma(N_l-R_{l\sigma,l\sigma})\mu_l-\sum_{l\neq l'\sigma\sigma'}T_{l\sigma,l'\sigma'}\mu_{l'}$, where N_l is the number of propagating channels per spin for the lead l, $T_{l\sigma,l'\sigma'}(R_{l\sigma,l'\sigma'})$ is the transmission (reflection) amplitude from the σ'-spin channel of lead l' to the σ-spin channel of the lead l, and μ_l is the chemical potential of the reservoir attached to the lead l. The Hall conductance is defined as $G_{\rm H}=-r_{yx}/(r_{xx}^2+r_{yx}^2)$, where $r_{yx}=(\mu_2-\mu_4)/I_1$ and

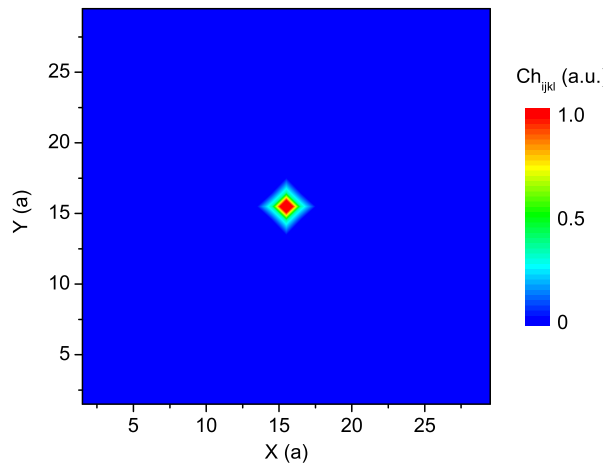

FIG. 2: Spatial configuration of the spin chirality defined by Eq. (3). System size is set to $30a\times30a$. Parameters are $S=1$ and $\theta=0.3\pi$. Strong spin chirality is seen near the center.

$r_{xx}=(\mu_1-\mu_3)/I_1$ are the Hall resistance and the resistance, respectively. The spin Hall conductance is defined as $G^\nu_{\rm sH}=(I^\nu_{2+}-I^\nu_{2-})/(\mu_1-\mu_3)$, where $I^\nu_{2\pm}$ is the current in the lead 2 with a polarization in the $\pm\nu\,(\nu=X,Y,Z)$ direction.

The uppermost panel of Fig. 3 shows the Hall conductance as a function of the energy of the conduction electrons and the angle of the local spin θ for $JS=1.0\,t$ in a system of size $30a\times30a$. The Hall conductance is non zero for $\theta\neq0,\pi/2,\pi$. Its sign changes when the sign of S_z changes. The amplitude of the Hall conductance oscillates with the energy. This is because the Hall effect is proportional to the momentum of the x-direction, hence it decreases when the Fermi energy is close to the energy where new propagating channels open. We note that the (charge) Hall effect also induces spin current density to be polarized parallel to the local spins near the interface between the lead 2 (or 4) and the sample.

If the Fermi energy is comparable to the exchange coupling energy, the adiabatic approximation that neglects the minority spin components cannot be applied, and we expect a spin current with a polarization that is not parallel to the local spins. To confirm this, we plot the spin Hall conductance for each polarization direction in the lower 3 panels of Fig. 3. $G^{Y,Z}_{\rm sH}$ vanish at $\theta=0,\pi$ while $G^X_{\rm sH}$ vanishes at $\theta=0,\pi/2,\pi$. At $\theta=\pi/2$, the local spins near the interface between the sample and the lead 2 almost direct in the x-direction, and the suppression of the Hall conductance results in a suppression of $G^X_{\rm sH}$. The non-vanishing Y- and Z- components of the spin Hall conductance at $\theta=\pi/2$ do not induce a voltage drop in

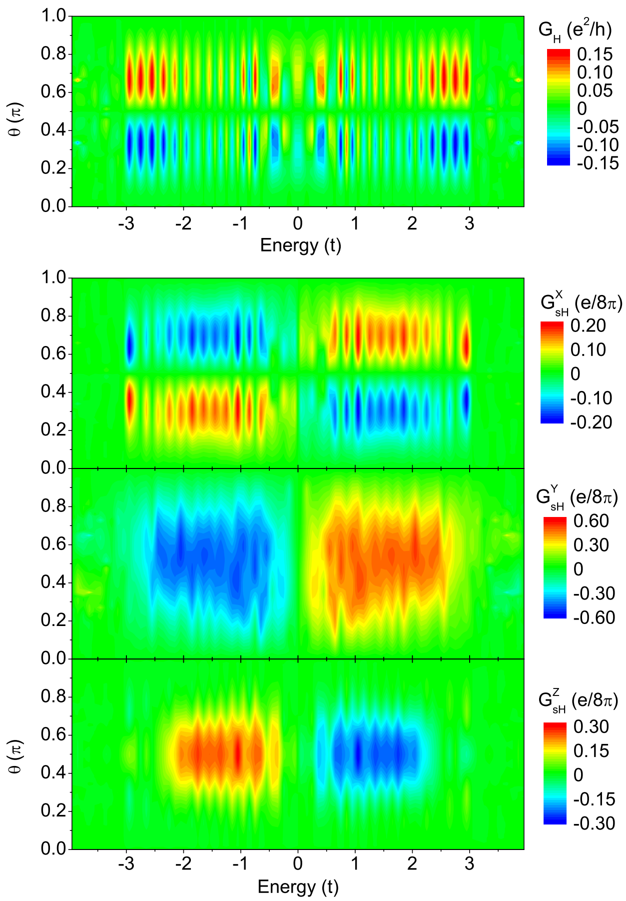

FIG. 3: Hall conductance and spin Hall conductance for each polarization: Exchange coupling constant is $JS = t$, and the system size $30a \times 30a$. Hall conductance and x-component of the spin Hall conductance disappear at $\theta = 0, \pi/2, \pi$, where the spin chirality vanishes.

the transverse direction like the spin Hall effect predicted in the spin-orbit system [16, 17]. The direction of polarization rotates while electrons propagate in the sample due to the precession induced by the exchange coupling. This precession is an important feature of the mesoscopic spin Hall effect that is also obtained in a 2-dimensional electron system with spin-orbit interaction. In contrast, only Z-component of the spin current is expected in bulk spin-orbit systems.

To make contact with the 2-dimensional system with spin-orbit interaction, we consider the Rashba spin-orbit interaction represented as

$$H = -t \sum_{<i,j>,\sigma,\sigma'} V_{i\sigma,j\sigma'} c^{\dagger}_{i\sigma} c_{j\sigma'} \tag{4}$$

with

$$V_{i+\hat{\mathbf{x}},i} = \begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix}, \tag{5}$$

and

$$V_{i+\hat{\mathbf{y}},i} = \begin{pmatrix} \cos\gamma & -\mathrm{i}\sin\gamma \\ -\mathrm{i}\sin\gamma & \cos\gamma \end{pmatrix}, \tag{6}$$

where γ is the coupling strength of the Rashba spin-orbit interaction [19, 20, 21]. The parameters JS and γ can be regarded as a gauge field strength [22, 23]. Fig. 4 shows the dependence on the coupling strength of the spin Hall conductances both for the chiral spin system and for the spin-orbit system. The spin Hall conductances oscillate with the coupling constant in both cases. Indeed, the component of the spin Hall conductance in the spin-orbit system shows spin precession by changing the length of the lead where the spin-obit interaction is present [24]. This oscillation cannot be obtained in bulk system with a spin-orbit interaction and a local electric field, where a monopole in momentum space description is possible. In this sense, the spin Hall current obtained in the present paper should be distinguished from the bulk spin Hall effect described by the Kubo formula. We also show the absolute value of the spin Hall conductance $|G_{\rm sH}| = \sqrt{{G^X_{\rm sH}}^2 + {G^Y_{\rm sH}}^2 + {G^Z_{\rm sH}}^2}$ in the weak coupling regime. Here, both spin Hall conductances show linear dependence on the coupling constant in weak coupling regime.

For measuring the present Hall effect in actual systems, an experimental setup using ferromagnetic semiconductors, such as (Ga,Mn)As, can be used. The proposed vortex spin configuration [25] can be obtained in dilute magnetic semiconductors with low Curie temperatures

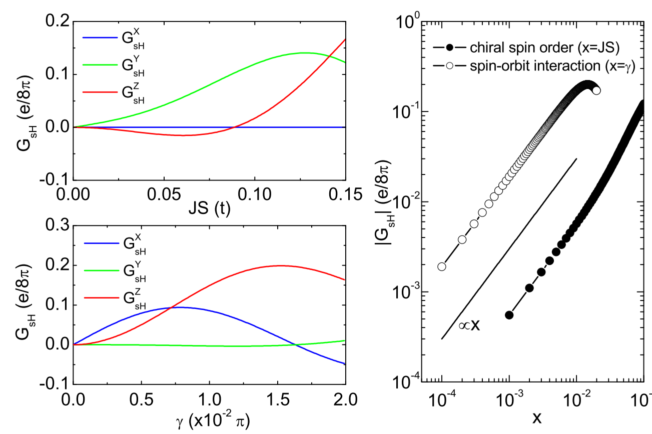

FIG. 4: Coupling parameter dependences of the spin Hall conductances of the chiral spin order system with $\theta = \pi/2$ (left-upper panel) and of the Rashba spin-orbit interaction system (left-lower panel). The right panel shows the log-log plot of absolute values of spin Hall conductances in a weak coupling regime. Both spin Hall conductances show a linear dependence on the coupling parameters JS and γ.

[26, 27]. Because of the small saturation magnetization (≈ 0.01 T) of magnetic semiconductors, the coupling energy between the conduction spin and local magnetic moment should be comparable to the Fermi energy ($\approx$ meV) [13]. For our calculations, by setting the tight binding parameter $a = 10\,\mathrm{nm}$ and $m^* = 0.05\,m_e$, the corresponding exchange energy becomes $J \approx 6.9\,\mathrm{meV}$. θ should be adjusted approximately to $\pi/2$ to minimize heating effect that destroys the spin order.

In conclusion, we have investigated a new mesoscopic Hall effect driven by a local spin system with spin chirality, which might be experimentally detected in 2DES embedded in ferromagnetic semiconductors. The local spin system is assumed to have the geometry of a vortex with a chirality at the center. We have predicted a Hall effect, which induces both a charge and a spin Hall conductance. Our numerical results based on the Landauer-Buttiker formula and the recursive Green function technique show that a voltage drop is obtained in the presence of spin chirality. No uniform electric field is required inside the sample. We have pointed out that the present Hall effect is related to the spin Hall effect obtained for a 2-dimensional spin-orbit system, but should be distinguished from the usual bulk spin Hall effect driven by monopoles in momentum space described by the Kubo formula.

The effect of randomness on the charge and the spin Hall effect, which is a problem left for the future, is also interesting. For $\theta = \pi/2$, the Hamiltonian becomes a real matrix via the unitary transformation $U = \exp(-\mathrm{i}\pi\sigma_x/4)$, and the system is mapped to an orthogonal system with a spin Hall current.

The authors are grateful to M. Yamamoto, S. Kettemann and Y. Avishai for valuable discussions. This work has been supported by the Deutsche Forschungsgemeinschaft via SFBs 508 and 668 of the Universität Hamburg, and by the European Union via the Marie-Curie-Network MCRTN-CT2003-504574.

[1] Y. Taguchi, Y. Oohara, H. Yoshizawa, N. Nagaosa and Y. Tokura, Science **291,** 2537 (2001).
[2] S. Onoda and N. Nagaosa, Phys. Rev. Lett. **90,** 196602 (2003).
[3] G. Tatara and H. Kohno, Phys. Rev. B **67,** 113316 (2003).
[4] K. Ohgushi, S. Murakami and N. Nagaosa, Phys. Rev. B **62,** 6065(R) (2000),
[5] S. Murakami, N. Nagaosa and S. -C. Zhang, Science **301** 1348, (2003).
[6] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth and and A. H. MacDonald, Phys. Rev. Lett. **92,** 126603 (2004).
[7] Y. K. Kato, R. C. Myers, A. C. Gossard and D. D. Awschalom, Science **306** 1910, (2004).
[8] R. Landauer, IBM J. Res. Dev. **1,** 223 (1957).
[9] M. Büttiker, Phys. Rev. Lett. **57,** 1761 (1986).
[10] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995).
[11] P. Bruno, V. K. Dugaev and M. Taillefumier, Phys. Rev. Lett. **93,** 096806 (2004).
[12] P. Bruno, Phys. Rev. Lett. **93,** 247202 (2004).
[13] J. K. Furdyna, J. Appl. Phys. **64,** R29 (1988).
[14] T. Ando, Phys. Rev. B **44,** 8017 (1991).
[15] M. Yamamoto, T. Ohtsuki and B. Kramer, Phys. Rev. B **72,** 115321 (2005).
[16] L. Sheng, D. N. Sheng and C. S. Ting, Phys. Rev. Lett. **94,** 016602 (2005).
[17] B. K. Nikolić, L. P. Zârbo and S. Souma, Phys. Rev. B **72,** 075361 (2005),
[18] J. -I. Ohe, M. Yamamoto and T. Ohtsuki, Phys. Rev. B **68**, 165344 (2003).
[19] E. I. Rashba, Sov. Phys. Solid State **2,** 1109 (1960).
[20] T. Ando, Phys. Rev. B **40,** 5325 (1989).
[21] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78,** 1335 (1997).
[22] I. L. Aleiner and V. I. Fal'ko, Phys. Rev. Lett. **87,** 256801 (2001).
[23] I. L. Aleiner and V. I. Fal'ko, Phys. Rev. Lett. **89,** 079902(E) (2002).
[24] J. -I. Ohe, T. Ohtsuki and S. Murakami, (unpublished).
[25] T. Shinjo, T. Okuno, R. Hassdorf, K. Shigeto and T. Ono, Science **289,** 930 (2000).
[26] T. Dietl, J. König and A. H. MacDonald, Phys. Rev. B **64,** 241201(R) (2001).
[27] A. M. Nazmul, S. Sugahara and M. Tanaka, Phys. Rev. B **67,** 241308(R) (2003).